DEPARTMENT OF PRE-UNIVERSITY EDUCATION

MODEL QUESTION PAPER FOR ANNUAL EXAMINATION APRIL-2022

II PUC

SUB: MATHEMATICS (35)

TIME: 3 Hours 15 MinutesMAX. MARKS: 100

Instructions :

(i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.

***(ii)* Use the graph sheet for the question on Linear programming in PART E.**

PART – A

Answer any TEN questions 10 X**1=10**

1. Give an example of a relation which is symmetric and transitive but not reflexive.

2. Define a binary operation.

3. Find the principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$.

4. If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, then find the value of x.

5. Define a row matrix.

6. Find the value of x if $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$.

7. If $y = e^{cosx},\ find\ \frac{dy}{dx}$.

8.If $y = sin(x^2+5),\ find\ \frac{dy}{dx}$.

9.Find $\int(2x^2+e^x)dx$.

10.Evaluate $\int_2^3 \frac{1}{x}dx$.

11.Find the unit vector in the direction of vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$.

12.Write two different vectors having same magnitude.

13.Write the direction cosines of x-axis.

14.Define feasible region of a linear programming problem.

15.Find P(A|B), if P(B) = 0.5 and P(A∩B) = 0.32.

PART- B

Answer any TEN questions 10 X2=20

16. Show that the signum function f:R→R given by $f(x) = \begin{cases} 1 & if \quad x > 0 \\ 0 & if \quad x = 0 \\ -1 & if \quad x < 0 \end{cases}$

is neither one-one nor onto.

17.Find the value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.

18.Write the domain and range of y =$\tan^{-1} x$.

19. Find the values of x, y and z from the equation $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$.

20. Find equation of line joining (1, 2) and (3, 6) using determinants.

21. If $x^2 + xy + y^2 = 100$, find $\frac{dy}{dx}$.

22. If $x = at^2$, $y = 2at$, $find \frac{dy}{dx}$.

23. Differentiate $\sin(cos(x^2))$ with respect to x.

24. Find the slope of tangent to curve $y = x^3 - x + 1$ at the point whose x-co-ordinate is 2.

25.Find $\int \frac{(logx)^2}{x}$dx.

26.Find $\int \frac{sin^2x - cos^2x}{sin^2x\, cos^2x}$ dx.

27.Evaluate $\int_0^{\frac{\pi}{2}} cos^2x\, dx$.

28.Find the order and degree of the differential equation $y^1 + y = e^x$.

29.Find the projection of the vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ on the vector $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$.

30.Find the area of the parallelogram whose adjacent sides are given by the vectors

$\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$.

31.Find the intercepts cut-off by the plane 2x+y-z=5.

32.Find the distance of the point (-6,0,0) from the plane 2x – 3y + 6z -2 = 0

33.The random variable X has a probability distribution P(X) of the following form where k is some number. Find the value of k.

$$P(X) = \begin{cases} k, & if\ x = 0 \\ 2k, & if\ x = 1 \\ 3k, & if\ x = 2 \\ 0, & otherwise \end{cases}$$

PART – C

Answer any TEN questions 10 X3=30

34. Show that the relation R in the set {1, 2, 3} given by R = { (1, 1), (2, 2), (3, 3), (1, 2), (2, 3)}

is reflexive but neither symmetric nor transitive.

35.Prove that $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$.

36. Find the inverse of the matrix $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$ using elementary operations.

37. Verify that the value of the determinant remains unchanged if its rows and columns are interchanged by considering third order determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$.

38. If $xy = e^{x-y}$ $find$ $\frac{dy}{dx}$.

39. If $x = a(cos\theta + \theta sin\theta)$, $y = a(sin\theta - \theta cos\theta)$ find $\frac{dy}{dx}$.

40.Verify mean value theorem, if $f(x) = x^2 - 4x - 3$ in the interval [a, b]

where a = 1 and b = 4.

41. Find the intervals in which the function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is

(a) increasing (b) decreasing.

42.Find $\int \frac{xe^x}{(1+x)^2}$dx.

43.Evaluate $\int \frac{1}{(x+1)(x+2)}\,dx$.

44.Evaluate $\int_0^2 e^x$dx as the limit of a sum.

45.Find the area of the region bounded by $y^2 = 9x$, x = 2, x= 4 and the x-axis

in the first quadrant.

46.Form the deferential equation representing the family of curves y=a.sin(x+b), where

a, b are arbitrary constants.

47.Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$.

48. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c} = \vec{o}$, find the value of

$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$.

49.Find x such that the four points A(3,2,1), B(4,x,5), C (4,2, -2) and D(6,5, -1) are co-planar.

50.Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$.

51.A man is known to speak truth 3 out of 4times. He throws a die and reports that it is a six.

Find the probability that it is actually a six.

PART –D

Answer any SIX questions **6X5=30**

52. Let A = R − {3} and B = R−{1}. Consider the function f: A → B defined $f(x) = \frac{x-2}{x-3}$.

Is f one-one and onto? Justify your answer.

53.Let f: N→R be a function defined as $f(x) = 4x^2 + 12x + 15$.

Show that f: N→S where, S is the range of f is invertible. Find the inverse of f.

54. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

(i) $(A+B)^T = A^T + B^T$ (ii) $(A-B)^T = A^T - B^T$.

55. Solve system of linear equation, using matrix method.

$2x + 3y + 3z = 5$
$x - 2y + z = -4$
$3x - y - 2z = 3$

56.If $y = 3\cos(logx) + 4\sin(logx)$ Show that $x^2y_2 + xy_1 + y = 0$.

57. The length x of a rectangle is decreasing at the rate of 5cm/minute and the width y is increasing at the rate of 4cm/minute.When x = 8 cm and y = 6cm find the rates of change of

a) the perimeter

b) the area of the rectangle.

58.Find the integral of $\frac{1}{x^2+a^2}$ with respect to x and hence evaluate $\int \frac{3x^2}{x^6+1}$ dx.

59. Find the area enclosed by the circle $x^2 + y^2 = a^2$ using integration.

60.Find the general solution of the differential equation $x\frac{dy}{dx} + 2y = x^2$ $(x \neq 0)$.

61.Derive the equation of a plane perpendicular to a given vector and passing through a given point both in vector and Cartesian form.

62.A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize

a) atleast once

b) exactly once

63.Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that

(i) the problem is solved (ii) exactly one of them solves the problem

PART – E

Answer any ONE question1 X10=10

64. (a) Maximise Z = 3x + 2y subject to the constraints x + 2y ≤ 10 , 3x + y ≤ 15, x,y ≥0

b) If the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = O$, when I is 2×2 identify matrix and O is 2×2 zero matrix. Using this equation find A^{-1}.

65. (a)Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$.

(b) Find the value of K so that the function $f(x) = \begin{cases} Kx + 1 \; if \; x \leq 5 \\ 3x - 5 if \; x > 5 \end{cases}$

is continuous at x = 5.

66. (a) Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

b) By using properties of determinants

Show that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$.

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